HBS: GREEN NEIGHBOURS IN CAMBRIDGE   Jan 2011

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CASE QUESTIONS:

1. What are the initial assumptions, advantages and disadvantages of the reciprocal energy switching model?
2. Using your estimates (from time series) for volatilities and correlation for rape oil and palm oil, which plant should George buy, and why?
3. Should George start with rape oil or palm oil, and when should he switch feedstocks?
4. Suppose George can raise half of the plant investment cost by issuing 50% of HBS equity for around $200,000. He plans modestly to buy a plant with capacity for 1000 units for a total investment cost of $350,000 if he decides on the fully flexible plant. Would you invest in the equity in this venture?

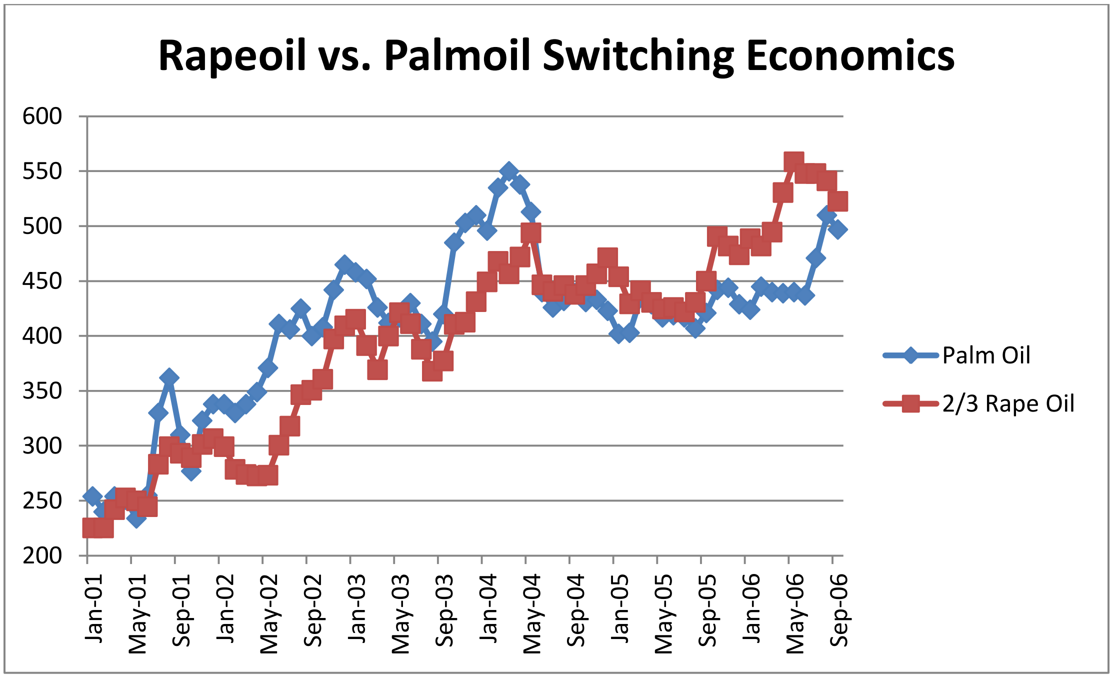
"In February 2004, Harvard University in Cambridge, Massachusetts, announced that it had begun to use B20 in all of its diesel vehicles and equipment, including shuttle buses, mail trucks and solid waste and recycling trucks. Although a number of alternative fuels were studied, biodiesel was finally selected because it provided the greatest health and environmental benefits in the most cost-effective way, according to David Harris Jr., general manager of transportation services at Harvard. But there were other reasons for the switch as well. 'Harvard is not a stand-alone campus,' Harris said. 'Our shuttle buses drive down the streets of Cambridge, past houses and other schools. We feel a responsibility to be a good neighbour and be as environmentally friendly as possible. Biodiesel helps us accomplish that using the vehicles we already have'." (Pahl, pp. 278-279) World Energy Alternatives LLC (of Massachusetts) supplied the biodiesel. Pahl believes their success and stability is partly due to "feedstock flexibility". (Pahl, p. 224).

George Gamble graduated from Harvard Business School in September 2006, and believed he could make a worthwhile contribution to his alma mater and to his own modest wealth by constructing and operating a flexible biodiesel plant in Cambridge to supply World Energy (and Harvard) with a sustainable, renewable, environmentally friendly transportation fuel, using a flexible production plant. George founded Harvard Biodiesel Sustainable (HBS).

He believes World Energy might guarantee a fixed price of $80 per unit for the biodiesel output. (One unit is 1/10 of one ton, but George believes a new venture scale plant might produce 1000 units, or 100 tons.) He plans to import canola (rape) oil from Canada, or palm oil from Malaysia, for use in a plant, capable of switching between rape oil and palm oil. The equivalent (2/3 canola=one unit of palm oil) cost of both canola oil and palm oil is currently around $40 per unit, efficiency is assumed to be 100%, there are no other operating costs apart from feedstock, and switching costs are about $20 per unit for switching either way from canola to palm oil. HBS is registered as a charity, and so pays no tax.

George is convinced that the facility to switch basic feedstock inputs is critical, after examining the relative prices over the last five years as shown in Figure 1.

Figure 1



Seemans A.G. has offered to supply a completely flexible plant for $350 per unit of production, with an infinite life. But there is a dispute on whether switching costs are variable as in Table 1, or constant as in Table 2.

A little rusty on switching options, George turned to some classical finance teachers, Professor Marshall at the University of Cambridge and Professor Jevons at the University of Manchester. Use net present values ("deterministic"), said Professor Marshall, because that is a trusted and established method. Wait, said an up and coming American finance Professor Brash. Marshall assumes certainty in feedstock prices. As you know both palm oil and rape seed oil prices are variable, and cultivation geographically distance, so use the new Adkins and Paxson (2011) approach ("stochastic X_1 and X_2"). What a palaver, thought George, that these academics cannot agree. What difference does it make anyhow?

Here is Professor Brash's story.

The spot price X_I for feedstock $I \in \{1,2\}$ is assumed to follow a geometric Brownian motion process with drift:

$$dX_I = \alpha_I X_I dt + \sigma_I X_I dz_I \tag{1}$$

where α_I is its instantaneous drift rate, σ_I is the known instantaneous volatility rate, and dz_I is the increment of a standard Wiener process. Dependence between the two spot price variables is described by the instantaneous covariance term $\rho\sigma_1\sigma_2$ where $Cov\left[dX_1, dX_2\right] = \rho\sigma_1\sigma_2 X_1 X_2 dt$ and $|\rho| \le 1$. For each feedstock, the spot price is adjusted by the conversion rate, so that the valuation relationship is expressed in terms of one unit of output.

The function F_I for $I \in \{1,2\}$ is defined as the plant value from using feedstock I together with the embedded switching option to replace the incumbent by the substitute feedstock. The value of F depends not only on the price of the feedstock that is currently in use but also the price of the substitute feedstock, so $F_I = F_I\left(X_1, X_2\right)$. Standard contingent claims analysis can be applied to the plant value to determine its risk neutral valuation relationship, expressed for feedstock $I \in \{1,2\}$ by the partial differential equation:

$$\begin{aligned} &\tfrac{1}{2}\sigma_1^2 X_1^2 \frac{\partial^2 F_I}{\partial X_1^2} + \tfrac{1}{2}\sigma_2^2 X_2^2 \frac{\partial^2 F_I}{\partial X_2^2} + \rho\sigma_1\sigma_2 X_1 X_2 \frac{\partial^2 F_I}{\partial X_1 \partial X_2} \\ &\qquad + \theta_1 X_1 \frac{\partial F_I}{\partial X_1} + \theta_2 X_2 \frac{\partial F_I}{\partial X_2} - rF_I + \left(Y_0 - X_1\right) = 0. \end{aligned} \tag{2}$$

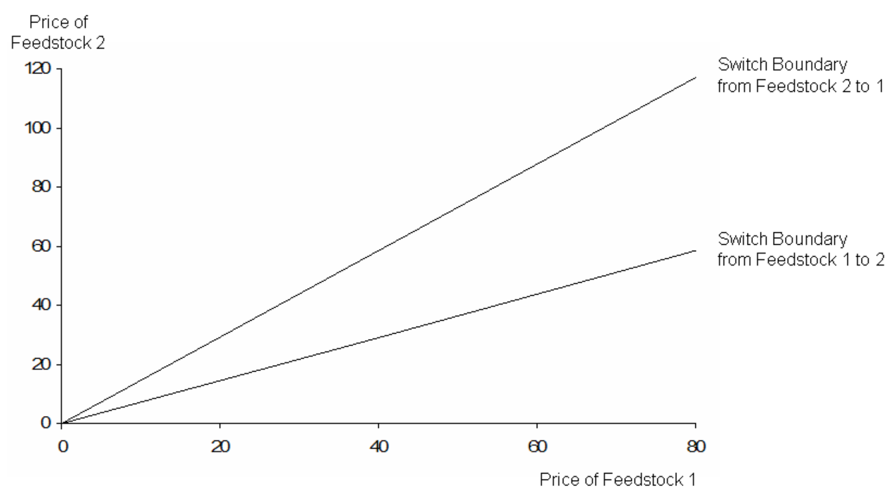
where r is the risk-free rate of interest, and θ_I denotes the risk-adjusted drift rates for feedstock $I \in \{1,2\}$. $r - \theta_I$ may be interpreted as the convenience yield for feedstock $I \in \{1,2\}$. Y_0 is the output price.

Representative discriminatory boundaries for the general switching model depicting the conditions favoring a viable switch from feedstock 1 to 2 and from feedstock 2 to 1 are illustrated in Figure 2. If the operator starts with feedstock 1 (the incumbent), below (lower right of) the switch boundary from feedstock 1 to 2, a switch from feedstock 1 to 2 is justified. The continuance region (don't switch) for the incumbent feedstock 1 is the complement of the switching boundary area. When the incumbent is feedstock 2, above (upper left of) the switch boundary from feedstock 2 to 1, the operator is justified in switching from feedstock 2 to 1. An easy model with a quasi-analytical solution assumes that the switching cost is variable, that is the cost for switching between the incumbent feedstock I and its substitute J is specified by:

$$K_{IJ} = k_I \, X_{IJ}^{\phi_I} \, X_{JJ}^{1-\phi_I} \;\; for\; I, J \in \{1,2\}, I \neq J \,, \tag{3}$$

where k_I and ϕ_I are known constant parameters. The switching cost is an increasing function of ϕ_I, which measures the relative significance of the two price levels in determining the switching cost. When $\phi_I = 1$, the switching cost depends only on the price of the incumbent feedstock and not on the price of the substitute. This parametric value is plausible for a multi-feedstock biofuel plant, if the switching cost is proportional to the prevailing price of the feedstock-in-use.

Figure 2
Timing Boundaries for the Variable Switching Cost Multiple Model



The solution of (2) takes the simplified form of (4), where the feedstock 1 is the incumbent:

$$F_1 = A_{14} X_1^{\beta_{14}} X_2^{\eta_{14}} + \frac{Y_0}{r} - \frac{X_1}{r - \theta_1} \,, \tag{4}$$

where $\beta_{14} > 0$ and $\eta_{14} \le 0$. When feedstock 2 is the incumbent, the simplified form of the valuation function F_2 takes the form:

$$F_2 = A_{22} X_1^{\beta_{22}} X_2^{\eta_{22}} + \frac{Y_0}{r} - \frac{X_2}{r - \theta_2}, \tag{5}$$

where $\beta_{22} \le 0$ and $\eta_{22} > 0$.

For $I \in \{1,2\}$, where A_I, β_I and η_I are generic parameters whose values have to be determined, using as well the following characteristic equation for $I \in \{1,2\}$:

$$Q_I(\beta_I, \eta_I) = \tfrac{1}{2}\sigma_1^2 \beta_I(\beta_I - 1) + \tfrac{1}{2}\sigma_2^2 \eta_I(\eta_I - 1) + \rho\sigma_1\sigma_2\beta_I\eta_I + \theta_1\beta_I + \theta_2\eta_I - r = 0. \tag{6}$$

The remaining unknown parameters in (4) and (5) are determined by the conditions that have to be fulfilled at the instantaneous switching event. The value matching condition requires that at the optimal switching event the total plant value for the incumbent feedstock is equal to the value of switching, which is the difference between the total plant value for the substitute feedstock and the fixed investment cost required for the switch.

The two value matching relationships are expressed respectively as:

$$F_1(\hat{X}_{12}, \hat{X}_{22}) = F_2(\hat{X}_{12}, \hat{X}_{22}) - K_{12}, \tag{7}$$

$$F_2(\hat{X}_{11}, \hat{X}_{21}) = F_1(\hat{X}_{11}, \hat{X}_{21}) - K_{21}, \tag{8}$$

for $\hat{X}_{12} > \hat{X}_{22}$ and $\hat{X}_{11} < \hat{X}_{21}$. From (4) and (5), the value matching relationships (7) and (8) become respectively:

$$A_{14}\hat{X}_{12}^{\beta_{14}}\hat{X}_{22}^{\eta_{14}} - \frac{\hat{X}_{12}}{r - \theta_1} = A_{22}\hat{X}_{12}^{\beta_{22}}\hat{X}_{22}^{\eta_{22}} - \frac{\hat{X}_{22}}{r - \theta_2} - K_{12}, \tag{9}$$

$$A_{22}\hat{X}_{11}^{\beta_{22}}\hat{X}_{21}^{\eta_{22}} - \frac{\hat{X}_{21}}{r - \theta_2} = A_{14}\hat{X}_{11}^{\beta_{14}}\hat{X}_{21}^{\eta_{14}} - \frac{\hat{X}_{11}}{r - \theta_1} - K_{21}. \tag{10}$$

The terms $\frac{X_1}{r - \theta_1}$ and $\frac{X_2}{r - \theta_2}$ specify the value of the cost of operating in perpetuity with feedstocks 1 and 2, respectively, when X_1 and X_2 represent the prevailing prices. The terms $A_{14}X_1^{\beta_{14}}X_2^{\eta_{14}}$ and $A_{22}X_1^{\beta_{22}}X_2^{\eta_{22}}$ denote, respectively, the value of the option to switch from the incumbent feedstock 1 to the substitute 2, and from the incumbent feedstock 2 to 1, when the incumbent is feedstock 1.

It is convenient if the underlying valuation relationships and implied value-matching conditions can be expressed solely as functions of the price ratio, which is feasible if the switching cost is proportional to the prevailing price of the feedstock-in-use as specified in (3). The price ratios along the two discriminatory boundaries are denoted by $\hat{W}_{12}$ and $\hat{W}_{21}$, where: $\hat{W}_{IJ} = \frac{\hat{X}_{IJ}}{\hat{X}_{JJ}} > 1, \, for \; I, J \in \{1, 2\}, I \neq J$. The quantity $\hat{W}_{IJ}$ specifies the price ratio at which the incumbent feedstock I is replaced by the substitute J. Rewriting the value-matching relationship for the switching model if there are variable switching costs (9) and (10) become, respectively:

$$A_{14}\hat{W}_{12}^{\beta_{14}} - \frac{\hat{W}_{12}}{r-\theta_1} = A_{22}\hat{W}_{12}^{\beta_{22}} - \frac{1}{r-\theta_2} - k_1\hat{W}_{12}^{\phi_1}, \tag{11}$$

$$A_{22}\hat{W}_{21}^{1-\beta_{22}} - \frac{\hat{W}_{21}}{r-\theta_2} = A_{14}\hat{W}_{21}^{1-\beta_{14}} - \frac{1}{r-\theta_1} - k_2\hat{W}_{21}^{\phi_2}. \tag{12}$$

Their smooth-pasting conditions are respectively:

$$\beta_{14}A_{14}\hat{W}_{12}^{\beta_{14}-1} - \frac{1}{r-\theta_1} = \beta_{22}A_{22}\hat{W}_{12}^{\beta_{22}-1} - \phi_1 k_1\hat{W}_{12}^{\phi_1-1}, \tag{13}$$

$$(1-\beta_{22})A_{22}\hat{W}_{21}^{-\beta_{22}} - \frac{1}{r-\theta_2} = (1-\beta_{14})A_{14}\hat{W}_{21}^{-\beta_{14}} - \phi_2 k_2\hat{W}_{21}^{\phi_2-1}. \tag{14}$$

The implied Q function (6) has closed-form solutions for β_{14} and β_{22}:

$$\beta_{14} = \tfrac{1}{2} - \frac{(\theta_1-\theta_2)}{\sigma_H^2} + \sqrt{\left(\tfrac{1}{2} - \frac{(\theta_1-\theta_2)}{\sigma_H^2}\right)^2 + \frac{2(r-\theta_2)}{\sigma_H^2}}, \tag{15}$$

$$\beta_{22} = \tfrac{1}{2} - \frac{(\theta_1-\theta_2)}{\sigma_H^2} - \sqrt{\left(\tfrac{1}{2} - \frac{(\theta_1-\theta_2)}{\sigma_H^2}\right)^2 + \frac{2(r-\theta_2)}{\sigma_H^2}}, \tag{16}$$

where $\sigma_H^2 = \sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$.

The values for the unknown quantities [W_{12}, W_{21}, A_{14} and A_{22}] are determined by using the closed-form solutions [15-16] for β_{14} and β_{22}, and then solving the four equations [11-14] for the four unknowns. Since $W_{12}=X_{12}/X_{22}$ and $W_{21}=X_{21}/X_{11}$, assuming values for $X_{21}=X_{22}$ are observable, the values for X_{12} and X_{11} are easily derived. Figure 3 shows that these results can be easily calculated using Excel, solving the four equations simultaneously using Solver.

The values for the various unknown quantities, which are evaluated from the data in Tables 1 and 2, using equations (11)-(16) are shown in Figure 3. The corresponding switching values for the feedstock 2 price given the feedstock 1 prices, $\hat{X}_{21} = 40.0$ and $\hat{X}_{22} = 40.0$, are X_{11}=27.3307 and X_{12}= 54.7204.

Table 1
Parametric Values for the Switching Model
Based on Variable Switching Costs

k_1	k_2	ϕ_1	ϕ_2
0.5	0.5	1.0	1.0

Table 2

Representative Data for the General Switching Model

Convenience Yield for feedstock 1	θ_1	5%
Convenience Yield for feedstock 2	θ_2	3%
Volatility for feedstock 1	σ_1	20%
Volatility for feedstock 2	σ_2	25%
Risk-free interest rate	r	7%
Constant Switching Cost		
From feedstock 1 to 2	K_{12}	20
From feedstock 2 to 1	K_{21}	20
Correlation feedstocks 1 and 2	ρ	.5

Figure 3

	A	B	C	D	E	F	G	H	I
1	**MULTIPLE: VARIABLE SWITCHING COST**								
2		Input							
3	k1	0.5							
4	k2	0.5							
5	$\phi 1$	1							
6	$\phi 2$	1							
7	K12	20							
8	K21	20							
9	$\sigma 1$	20.0%							
10	$\sigma 2$	25.0%							
11	ρ	0.5							
12	r	7%							
13	$\theta 1$	5%							
14	$\theta 2$	3%							
15	$r-\theta 1$	2%							
16	$r-\theta 2$	4%							
17	$\sigma 12$	5.25%							
18	X21	40.00							
19	X22	40.00							
20	Solution								
21	X11	**27.3307**							
22	X12	**54.7204**							
23	$\beta 14$	1.3592	EQ 15						
24	$\beta 22$	-1.1211	EQ 16						
25	W12	1.3680							
26	W21	1.4636							
27	A14	30.4441							
28	A22	5.5318							
29	VM 1	0.0000	EQ 11						
30	VM 2	0.0000	EQ 12						
31	SP 1	0.0000	EQ 13						
32	SP 2	0.0000	EQ 14						
33	SUM	0.0000							
34	Solver B33=0, Changing B25:B28								
35									
36	EQ 11	B27*(B25^B23)-B25/B15-(B28*(B25^B24)-1/B16-B3*(B25^B5))							
37	EQ 12	B28*(B26^(1-B24))-B26/B16-(B27*(B26^(1-B23))-1/B15-B4*(B26^B6))							
38	EQ 13	B23*B27*(B25^(B23-1))-1/B15-(B24*B28*(B25^(B24-1))-B5*B3*(B25^(B5-1)))							
39	EQ 14	(1-B24)*B28*(B26^(-B24))-1/B16-((1-B23)*B27*(B26^(-B23))-B6*B4*(B26^(B6-1)))							
40	SUM	SUM(ABS(B29:B32))							
41									
42	X11	B18/B26							
43	X12	B25*B19							
44	$\beta 14$	0.5-(B13-B14)/B17+SQRT((0.5-(B13-B14)/B17)^2+2*(B12-B14)/B17)							
45	$\beta 22$	0.5-(B13-B14)/B17-SQRT((0.5-(B13-B14)/B17)^2+2*(B12-B14)/B17)							
46									
47	Y0	80							
48	Suppose X1=X2=40 at start			Option	Production Cost				
49	F1	EQ 4	**360.62**	**1217.76**	**-857.14**				
50	F2	EQ 5	**364.13**	**221.27**	**142.86**				
51	Since F2>F1, start with X2, more volatile, but higher conyield, so lower perpetual production cost.								
52	Switching option is greater for X1, but not enough to offset negative production cost.								
53									
54	F1	EQ 4	B27*(B18^B23)*(B19^(1-B23))+B47/B12-B18/B15						
55	F2	EQ 5	B28*(B18^B24)*(B19^(1-B24))+B47/B12-B19/B16						

Multiple Switching, Constant Switching Costs

For multiple reciprocal (back and forth) switching, the solution for (2) is derived in Adkins and Paxson (2011). "Wow", said George in trying to read this article: "only rocket scientists can operate this facility, unless Seemans confirms that the switching cost is variable". "No problem" says Professor Brash, "these equations are just to impress other professors. I will provide for a special ($10000 per annum) fee an easier approximate decision rule, updated every month as convenience yields, expected volatilities and correlations of the feedstock change." Based on Table 2, a "good enough" approximate BRASH rule is:

X_{11}=-2.66 +.74 X_{21} EXAMPLE: X_{11}=-2.66 +.74 (40) = 26.94

X_{12}= 2.31+1.26 X_{22} EXAMPLE: X_{12}= 2.31+1.26 (40) = 52.71

The BRASH rule advises switching to feedstock 2 (if less than 26) if feedstock 1 is 40, and switching to feedstock 1 (if less than 40) if feedstock 2 is greater than 53, "pretty close" to the precise solution, and a lot easier on the operator's mind and common sense. George wondered if he should pay the fee, or whether HBS could use this approximate rule, even if volatilities and correlations change?

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